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Subject: Quarterly Status Report No. 6, Task Order NASr-63(07), NASA Hq. 80X0108(64), MRI Project 2760-P, "Nonlinear Dynamics of Thin Shell Structures," covering the period 15 July - 14 October 1965.

Gentlemen:

Please accept this letter as a quarterly progress report for the subject contract.

Many important physical systems can be described by the differential equation which represents a damped mass-spring system with control proportional to $By + Cy^3$ and driven by a force $D \cos(\omega t + \varphi)$. This equation is

$$y'' + Ay' + By + Cy^3 = D \cos(\omega t + \varphi) ,$$

$$A > 0 , \; y(0) = \alpha_o , \; y'(0) = \beta , \qquad (1)$$

which is the well-known Duffing equation.

In the usual situation, A and D are assumed small and a perturbation scheme is used to construct approximate solutions. In previous reports we developed an algorithm to construct rational approximations to the solution of (1). The validity of these approximations does not depend on the relative magnitudes of A and D.

In this quarterly report we investigate an example of type (1) for which the perturbation technique is impractical but for which our rational approximations are very effective. We also show how to extend the range of validity of our rational approximations by the method of analytical continuation.

Consider the equation

$$y'' + 0.2y' + 5y + 10y^3 = \cos \omega t \ , \ y(0) = 1 \ , \ y'(0) = 0 \ . \tag{2}$$

To cast this equation in the required form, set

$$y = 1 - 7t^2 v \ . \tag{3}$$

Then (2) becomes

$$\begin{aligned} &7t^2v'' + (28t + 1.4t^2)v' + (14 + 2.8t + 245t^2)v \\ &\quad - 1470t^4v^2 + 3430t^6v^3 - 15 + \cos \omega t = 0 \ , \\ &\quad v(0) = 1 \ . \end{aligned} \tag{4}$$

In (4), we replaced $\cos \omega t$ by a polynomial approximation which is accurate to five decimals for $0 \le \omega t \le 1$. Using our technique to obtain rational approximations to the solution of the resulting equation, we constructed rational approximations v_n to v and y_n to y where

$$y_n = 1 - 7t^2 v_n \ . \tag{5}$$

It is clear that the range of validity of our approximations is limited to the range of validity of the approximation to $\cos \omega t$.

In Tables I, II and III, the fourth and sixth order approximations to y for $\omega = 0$, 1 and 2 are listed. Also given are values determined by stepwise numerical integration which we call true. As is evident, the rational approximations are quite accurate. Since the accuracy of our approximation for $\cos \omega t$ is less than five decimals when $\omega t > 1$ and $\omega = 2$, we employed the analytic continuation technique for $\omega = 2$ to compute the approximate solutions for $\omega t > 0.8$. Thus the approximate solution was computed for $0 \le t \le 0.4$, and then the transformation $t = \tau + 0.4$ was utilized to convert (4) into a new initial value problem. Then the approximate solutions were evaluated for $\tau = 0(0.04)0.6$, i.e., $t = 0.4(0.04)1.0$.

In the final report we shall codify the results of the present period of investigation and include in it a FORTRAN program which computed these rational approximations to the solution of the generalized second order Ricatti equation.

Very truly yours,

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(25 copies of report submitted)

TABLE I

$\omega = 0$

t	$y(t)$ (true)	$y_4(t)$	$y_6(t)$
0.00	1.00000	1.00000	1.00000
0.04	0.98888	0.98888	0.98888
0.08	0.95625	0.95625	0.95625
0.12	0.90399	0.90399	0.90399
0.16	0.83481	0.83481	0.83481
0.20	0.75186	0.75185	0.75186
0.24	0.65838	0.65837	0.65838
0.28	0.55742	0.55739	0.55742
0.32	0.45162	0.45153	0.45162
0.36	0.34315	0.34290	0.34315
0.40	0.23373	0.23310	0.23373
0.44	0.12469	0.12329	0.12469
0.48	0.01708	0.01419	0.01708
0.52	-0.08822	-0.09375	-0.08821
0.56	-0.19037	-0.20036	-0.19033
0.60	-0.28848	-0.30558	-0.28837
0.64	-0.38151	-0.40951	-0.38127
0.68	-0.46826	-0.51230	-0.46774
0.72	-0.54734	-0.61415	-0.54628
0.76	-0.61722	-0.71527	-0.61516
0.80	-0.67629	-0.81588	-0.65482
0.84	-0.72298	-0.91619	-0.71633
0.88	-0.75596	-1.01640	-0.74481
0.92	-0.77421	-1.11668	-0.75632
0.96	-0.77720	-1.21720	-0.77346
1.00	-0.76495	-1.31808	-0.75084

TABLE II

$\omega = 1$

t	y(t) (true)	$y_4(t)$	$y_6(t)$
0.00	1.00000	1.00000	1.00000
0.04	0.98888	0.98888	0.98888
0.08	0.95625	0.95625	0.95625
0.12	0.90398	0.90398	0.90398
0.16	0.83478	0.83478	0.83478
0.20	0.75179	0.75179	0.75179
0.24	0.65825	0.65824	0.65825
0.28	0.55718	0.55715	0.55718
0.32	0.45121	0.45113	0.45121
0.36	0.34251	0.34226	0.34251
0.40	0.23276	0.23215	0.23276
0.44	0.12328	0.12191	0.12328
0.48	0.01509	0.01225	0.01509
0.52	-0.09095	-0.09640	-0.09094
0.56	-0.19402	-0.20388	-0.19398
0.60	-0.29324	-0.31019	-0.29315
0.64	-0.38760	-0.41543	-0.38738
0.68	-0.47589	-0.51979	-0.47542
0.72	-0.55672	-0.62349	-0.55576
0.76	-0.62851	-0.72678	-0.62664
0.80	-0.68961	-0.82991	-0.68616
0.84	-0.73840	-0.93313	-0.73234
0.88	-0.77347	-1.03665	-0.76330
0.92	-0.79375	-1.14069	-0.77741
0.96	-0.79865	-1.24550	-0.77346
1.00	-0.78817	-1.35113	-0.75082

TABLE III

$\omega = 2$

t	y(t) (true)	$y_4(t)$	$y_6(t)$
0.00	1.00000	1.00000	1.00000
0.04	0.98888	0.98888	0.98888
0.08	0.95624	0.95624	0.95624
0.12	0.90396	0.90396	0.90396
0.16	0.83470	0.83470	0.83470
0.20	0.75160	0.75160	0.75160
0.24	0.65786	0.65786	0.65786
0.28	0.55647	0.55645	0.55647
0.32	0.45002	0.44996	0.45002
0.36	0.34063	0.34043	0.34063
0.40*	0.22992	0.22941	0.22992
0.44	0.11916	0.11916	0.11916
0.48	0.00931	0.00931	0.00931
0.52	-0.09882	-0.09882	-0.09882
0.56	-0.20448	-0.20448	-0.20448
0.60	-0.30682	-0.30682	-0.30682
0.64	-0.40484	-0.40486	-0.40484
0.68	-0.49732	-0.49740	-0.49732
0.72	-0.58279	-0.58304	-0.58279
0.76	-0.65955	-0.66024	-0.65955
0.80	-0.72579	-0.72745	-0.72579
0.84	-0.77969	-0.78327	-0.77969
0.88	-0.81959	-0.82666	-0.81959
0.92	-0.84421	-0.85710	-0.84423
0.96	-0.85280	-0.87472	-0.85286
1.00	-0.84524	-0.88031	-0.84540

*Point at which analytic continuation begins.